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THE PENETRATION EFFECT IN ANISOTROPIC STRATIFIED STRUCTURE WITH LOSSES AND FREQUENCY DISPERSION

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The stratified anisotropic slab with losses and frequency dispersion is considered. The penetration effect for parallel wave propagation along the interface is studied. Various variants (directions) of wave propagation are discussed in detail. The practical applications of the obtained results are proposed.

Keywords: *ANISOTROPIC STRUCTURE, PENETRATION EFFECT, REFLECTION COEFFICIENT, OPTICAL ISOLATOR, FREQUENCY DETECTOR.*

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1. INTRODUCTION

The stratified anisotropic structures are an important element of microwave engineering, terahertz electronics and optoelectronics [1-3]. Recently, they are also called as one-dimensional anisotropic photon crystals [4, 5] and widely discussed in the scientific literature from the point of view of their practical application.

The authors of [6, 7] have described the so-called penetration effect, when the wave, which propagates in dielectric along the interface with plane-parallel anisotropic medium, excites a bulk wave. Moreover, it is shown that the wave transmission coefficient through the plate in the considered case is not equal to zero without taking into account losses and frequency dispersion proper for real anisotropic materials.

The given paper is the continuation of works [6, 7]. Here we consider the possibility of the existence of the penetration effect for gyrotropic medium with losses and frequency dispersion. Moreover, in this work we consider in detail different types of wave incidence: from different sides of the plates and directions with regard to the anisotropy axis.

The main feature of the presented results from the physical point of view is the possibility of excitation of a bulk wave in the structure with frequency dispersion for parallel wave propagation along the interface both with and without losses. Practical application of the penetration effect can allow to decrease overall dimensions of optoelectronic devices in comparison with the already existing and to create new ones. According to the aforesaid, the aim of the present work is the analysis of construction principles of new optical devices based on anisotropic stratified structures.

2. STATEMENT OF THE PROBLEM

In the present work we consider the problem of wave reflection and transmission through stratified anisotropic structure with losses and frequency dispersion. Geometry of the problem is represented in Fig. 1. The relative magnetic permeability μ in each point of the structure is equal to 1, and the relative dielectric permittivity of each uniform layer is described by tensor in gyrotropic form [8]

$$\bar{\varepsilon} = \begin{vmatrix} \varepsilon_{xx} & -j\varepsilon_{xy} & 0 \\ j\varepsilon_{xy} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{vmatrix}. \quad (1)$$

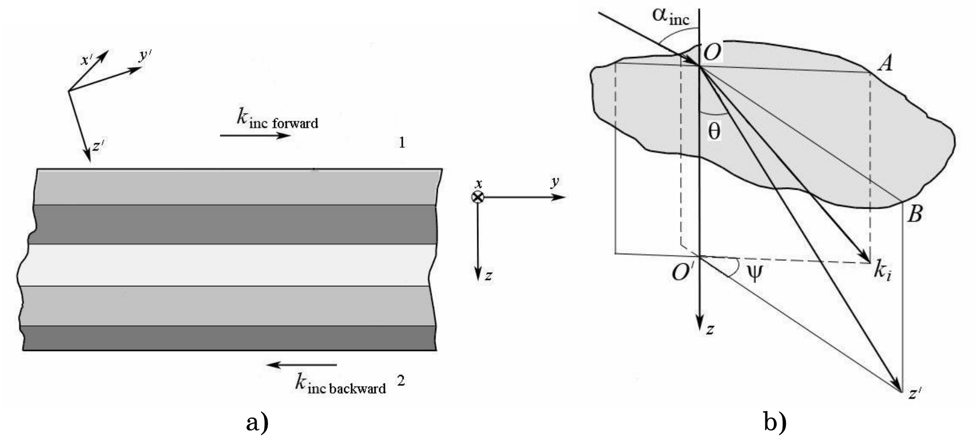

Fig. 1 *– The problem geometry*

Frequency dispersion and losses are taken into account in the Landau form [9]. In the specified case of the studied two-layered structure, dependence for ε_{xx} of each of two layers is given in Fig. 2. Elements ε_{xy} have the same behavior.

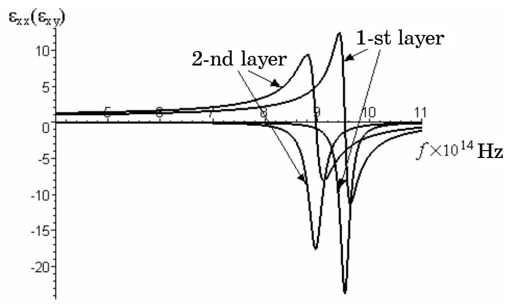

Fig. 2 *– Frequency dependence of the elements ε_{xx} and ε_{xy} of tensor (1)*

Direction of the anisotropy axis in the medium is chosen to be an arbitrary and the same for all layers (Fig. 1b); the incident wave propagates along the structure (Fig. 1a).

The main problem of the work is the study of the wave penetration effect into the structure with losses and frequency dispersion.

3. NUMERICAL RESULTS AND ANALYSIS

First of all, we will study the medium properties in the frequency band below the resonant frequencies [9] (f_{p1} = 900 THz and f_{p2} = 950 THz). In the range of $\omega < \omega_{p1}$, $\omega < \omega_{p2}$, real parts of the tensor elements of dielectric permittivity are positive and imaginary parts are negative. On the frequencies below 800 THz losses in both layers are negligible. On the frequencies from 800 THz to 850 THz only losses in the first layer are substantial. On the frequencies from 850 THz to 900 THz losses in both layers have a great influence.

The field structure in the considered slab is considerably defined by the properties of wave numbers. Their normal components in the uniform anisotropic layer for tangential incidence of a wave (wave propagation along the surface) are not equal to zero and are the solution of the same dispersion equation as for the medium without losses [7]

$$c_4 k_Z^4 + c_3 k_Z^3 + c_2 k_Z^2 + c_1 k_Z + c_0 = 0, \tag{2}$$

where

$$\begin{cases} c_4 = -\omega^2 \mu_0 \left(\varepsilon_{xx} \sin^2\theta + \varepsilon_{xx} \cos^2\theta\right) \\ c_3 = 2\omega^2 \mu_0 k_y \left(\varepsilon_{xx} - \varepsilon_{zz}\right) \sin\theta\cos\theta \\ c_2 = \omega^2\mu_0 \left[\omega^2\mu_0 \left(\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xx}^2 + \varepsilon_{xy}^2\right) + k_x^2\left(\varepsilon_{xx} - \varepsilon_{zz}\right)\right]\cos^2\theta + \\ \quad +\omega^2\mu_0\left[\omega^2\left(\varepsilon_{xx}\varepsilon_{zz} + \varepsilon_{xx}^2 - \varepsilon_{xy}^2\right) - \left(2k_x^2\varepsilon_{xx} + k_y^2\varepsilon_{xx} + k_y^2\varepsilon_{zz}\right)\right] \\ c_1 = 2\omega^2\mu_0 k_y\left[\omega^2\mu_0\left(\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xx}^2 + \varepsilon_{xy}^2\right) + \left(k_x^2 + k_y^2\right)\left(\varepsilon_{xx} - \varepsilon_{zz}\right)\right]\sin\theta\cos\theta \\ c_0 = -\omega^2\mu_0 k_y^2\left[\omega^2\mu_0\left(\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xx}^2 + \varepsilon_{xy}^2\right) + k_y^2\left(k_x^2 + k_y^2\right)\left(\varepsilon_{xx} - \varepsilon_{zz}\right)\right]\cos^2\theta + \\ \quad +\omega^6\mu_0\varepsilon_{zz}\left(\varepsilon_{xx} - \varepsilon_{xy}\right) + \omega^4\mu_0^2\left[k_x^2\left(\varepsilon_{xy}^2 - \varepsilon_{xx}\varepsilon_{zz}\right) - \varepsilon_{xx}\left(2\varepsilon_{zz}k_y^2 + \varepsilon_{xx}k_x^2\right)\right] + \\ \quad +\omega^2\mu_0\left(k_x^2 + k_y^2\right)\left(\varepsilon_{xx}k_x^2 + \varepsilon_{zz}k_y^2\right) \end{cases} \tag{3}$$

Here ω is the cyclic frequency; μ_0 is the magnetic permeability of free space; k_x, k_y, k_z are the components of the wave number; ε_{xx}, ε_{xy}, ε_{zz} are the components of tensor (1).

This implies the possibility of the existence of the wave penetration effect into anisotropic medium in the presence of losses. However, coefficients of this equation for the given case are complex quantities. Then z-components of the wave numbers are also complex in general case, i.e. wave is decaying along the z-axis, and, therefore, transmission is possible only at small layer thicknesses and taking into account the effect of multiple reflection [10].

Components of electromagnetic field are determined by the same correlations as for the medium without losses [6, 7] and they also depend on the mutual orientation of the anisotropy axis and direction of the wave incidence.

Results of the numerical calculations for the structure involving twelve two-layer periods are represented below. Fig. 3a corresponds to the incidence from one side of the structure (from medium 1 into medium 2), Fig. 3b – from another side (from medium 2 into medium 1), Thus, it is seen that such structure also possesses nonreciprocal properties. Darker regions correspond to the lower reflection coefficient.

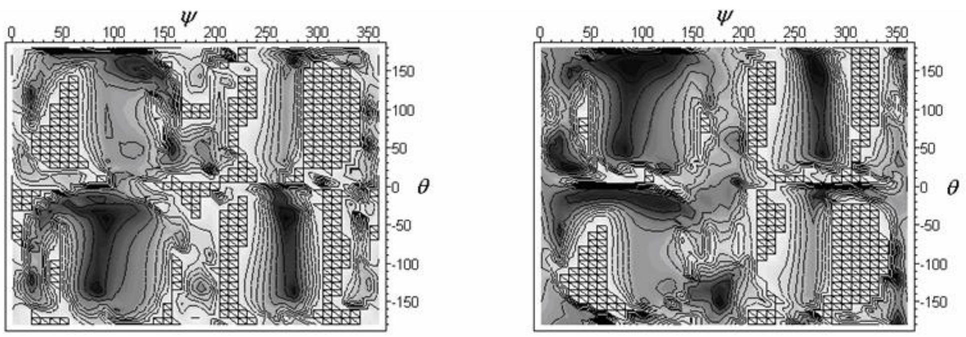

***Fig. 3** – Dependence of the reflection coefficients on the inclination angle of the anisotropy axis and the angle between the incidence plane and the plane containing anisotropy axis for twelve two-layer periods, d_2 = 100 nm, d_1 = 40 nm, f = 400 THz: direct transmission (a); backward transmission (b)*

We note that on the frequencies below 500 THz the losses are negligible, and the element ε_{xy} of tensor (1) is almost equal to zero. This means that the medium is not gyrotropic, but uniaxial anisotropic ($\varepsilon_{xx} \neq \varepsilon_{zz}$, $\varepsilon_{xy} = 0$). However, in this range one can observe the penetration effect, the total internal reflection, and the resonance peaks. Really, coefficients (3) in this range are not equal to zero, and, therefore, normal components of the wave vector are not equal to zero, too. Note, the resonance dependence is observed in isotropic structure as well [10], i.e. it is defined, first of all, by the phenomenon of multiple reflection, and not by anisotropy. Total internal reflection at any incidence angles also exists in uniform uniaxial mediums [9] that does not contradict the results obtained before. In the considered range, the nonreciprocal properties of the medium are also preserved. Indeed, though the wave numbers do not depend on the direction of the wave incidence (angle ψ), the field components are the functions of angle ψ.

We should note, the calculation method is violated at $\theta = 0°$ and $\theta = 180°$, when the anisotropy axis is perpendicular to the structure. Really, electromagnetic properties of gyrotropic medium are symmetrical with respect to the anisotropy axis [7]. In this case, reflection coefficient does not depend on the angle ψ (see Fig. 3). Here the wave penetration effect does not vanish, since normal components of the wave number are not equal to zero. Indeed, k_z are found as the solutions of biquadratic equation (4)

$$c_4 k_Z^4 + c_2 k_Z^2 + c_0 = 0, \quad (4)$$

where

$$\begin{cases} c_4 = -\omega^2 \mu_0 \varepsilon_{zz} \\ c_2 = 2\omega^4 \varepsilon_{zz} \mu_{zz} \left(\varepsilon_{xx}\mu_{xx} + \varepsilon_{xy}\mu_{xy}\right) - \omega^2 \left(k_x^2 + k_y^2\right)\left(\varepsilon_{xx}\mu_{zz} + \varepsilon_{zz}\mu_{xx}\right) \\ c_0 = \omega^6 \mu_{zz}\varepsilon_{zz}\left[\left(k_{xx}^2 - k_{xy}^2\right)\left(\varepsilon_{xy}^2 - \varepsilon_{xx}^2\right)\right] + \omega^4\left(k_x^2 + k_y^2\right) \\ \quad \left[\varepsilon_{xx}\varepsilon_{zz}\left(\mu_{xx}^2 - \mu_{xy}^2\right) + \mu_{xx}\mu_{zz}\left(\varepsilon_{xx}^2 - \varepsilon_{xy}^2\right)\right] - \omega^2 \varepsilon_{xx}\mu_{xx}\left(k_x^2 + k_y^2\right)^2 \end{cases} \quad (5)$$

Here μ_{xx}, μ_{xy}, μ_{zz} are the magnetic permeability tensor components.

In Fig. 4 we present the dependence of the reflection coefficients R on the frequency of 700 THz, where losses are not big yet but real parts of the tensor elements increase. Dependence of R on the frequency of 850 THz, where losses have a great influence, is shown in Fig. 5. Material parameters of layers correspond to the range below the resonant frequency (ω_{p1} = 950 THz, ω_{p2} = 900 THz).

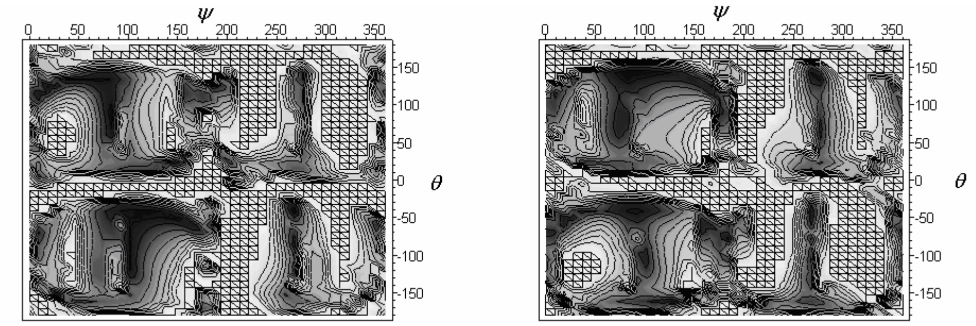

Fig. 4 *– Dependence of the reflection coefficients on the inclination angle of the anisotropy axis and the angle between the incidence plane and the plain containing anisotropy axis for twelve two-layer periods,* d_2 = 100 *nm,* d_1 = 40 *nm,* f = 700 *THz: direct transmission (a); backward transmission (b)*

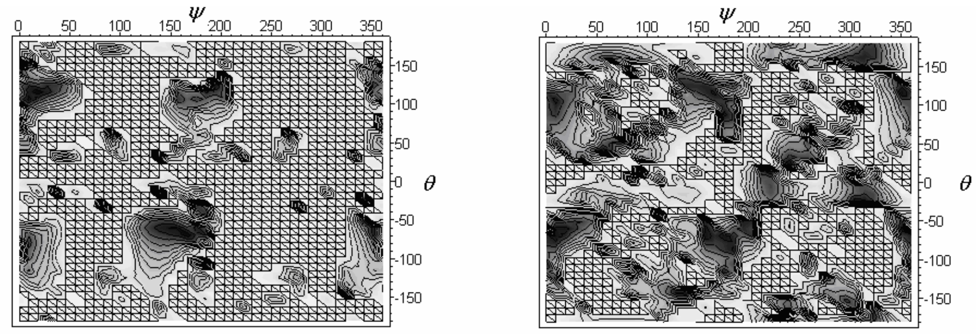

Fig. 5 *– Dependence of the reflection coefficients on the inclination angle of the anisotropy axis and the angle between the incidence plane and the plain containing anisotropy axis for twelve two-layer periods,* d_2 = 100 *nm,* d_1 = 40 *nm,* f = 850 *THz: direct transmission (a); backward transmission (b)*

It is seen from the figures that increase in the losses leads to the increase in the dependence of the reflection coefficient on the angles, i.e. to the increase in the selective properties over angles. However, in this case minimums of the reflection coefficients decrease. To increase minimums it is necessary to decrease the layer thickness, but this decrease is restricted by the current technological possibilities.

In the range of $\omega < \omega_{p1}$, $\omega > \omega_{p2}$, tensor elements of the dielectric permittivity for the first layer are positive, for the second are negative. Numerical calculations show that the penetration effect in this range at the specified parameters also exists.

In the range of $\omega > \omega_{p1}$ and $\omega > \omega_{p2}$, all tensor elements of the dielectric permittivity of two layers, both real and imaginary, are negative. Numerical calculations confirm the existence of the penetration effect in this range at the specified parameters. However, resonance peaks are observed only for the wave transmission from medium 2 into medium 1. Reflection coefficient at the wave transmission from medium 1 into medium 2 is sufficiently large and is the continuous frequency function.

The performed analysis shows the possibility of the application of these structures in optical electronics taking into account losses and frequency dispersion of the materials. In Fig. 6 we represent the frequency dependences of the reflection coefficient for the foregoing parameters of the structure and angles specified in the figure. Here we consider four variants of the plane harmonic wave transmission from medium 1 into medium 2 and inversely.

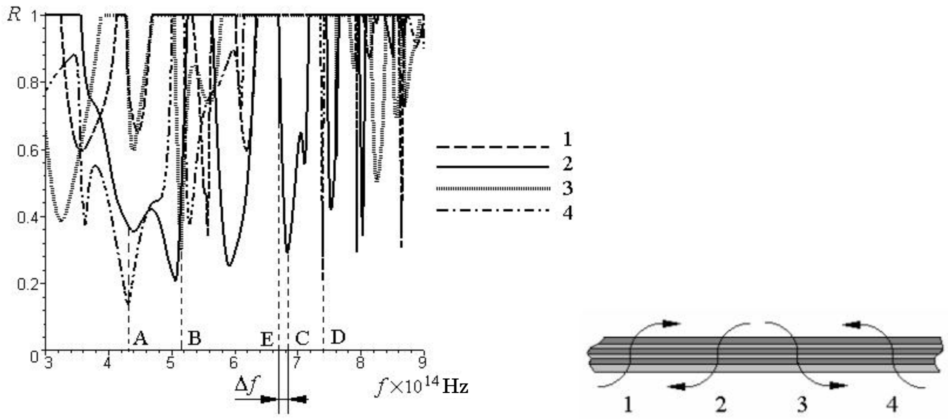

***Fig. 6** – Frequency dependence of the reflection coefficient at* $\theta = 21°$, $\psi = 169°$

Let us analyze the obtained results. On the frequency of 432 THz (point A) transmission coefficient from medium 2 into medium 1 along the negative direction of the y-axis is equal to 0,16; and from medium 1 into medium 2 along the same direction – 0,38. Thus, at the wave incidence from the negative direction of the y-axis, a modulated wave will propagate along the structure. This wave is described as

$$s(t) = A_1 \sin\left(\omega t - ky + \delta_1\right) + A_1 T_{12} \sin\left(\omega t - ky + \delta_2\right) + A_1 T_{12} T_{21} \sin\left(\omega t - ky + \delta_3\right) + \ldots, \quad (6)$$

where R_{12}, R_{21} are the absolute values of the reflection coefficients for the wave transmission from medium 1 into medium 2 and from medium 2 into medium 1; A_1 is the constant determined by the source power; $T_{12} = \sqrt{1 - R_{12}^2}$ is the transmission coefficient from medium 1 into medium 2; $T_{21} = \sqrt{1 - R_{21}^2}$ is the transmission coefficient from medium 2 into medium 1; k is the wave number in mediums 1 and 2; δ_1, δ_2, δ_3 are the phase shifts at the wave transmission through the anisotropic structure.

Here, if multiple refracted waves add in phase, the interference maximums will be observed; and if multiple refracted waves add in antiphase – the interference minimums.

On the frequency of 740 THz (point D) reflection coefficient for the direction 4 is equal to 0,2, for other directions – to 1. Thus, the structure possesses valve properties and can be practically applied. The same structure properties are on the frequency of 682 THz (point C) for the wave incidence from medium 1 into medium 2 from the negative direction of the y-axis. On the frequency of 516 THz (point B) reflection coefficient in the direction 3 is equal to 0,23, in the direction 2 – to 0,4.

Principle of operation of analog frequency detectors in the optical range based on the stratified anisotropic structures is described in [6, 7]. In the medium with losses we will consider, for example, the range 670-682 THz (interval EC). On this region of the amplitude-frequency characteristic, frequency dependence of the reflection coefficient modulus can be approximately considered as linear one for the direction 2. Therefore, reflection coefficient should be changed by the same law that the frequency does.

Thus, frequency change by the law of the transmitted message will lead to the change of the signal amplitude by the same law. We note that reflection coefficient for the rest of three directions in this range is equal to 1. That is the wave reflected from medium 2 along the direction 1 will not get back to the transmitting path. Moreover, wave will not be re-reflected from medium 1 into medium 2 and back and create spurious modulation, since reflection coefficient from the direction 4 is equal to 1. Total reflection of the wave from the direction 3 can be considered as the disadvantage, i.e. from this direction reflected wave will get back to the transmitting path. Thus, using the specified range in the medium 1 it is necessary to place the matched load on the transmitting path output. The second solution of this problem is the matching of the structure parameters in such a way that reflection from the direction 3 in the given range was minimal.

4. TYPES OF THE DEVICES BASED ON THE PENETRATION EFFECT

Principle of the construction of optical devices based on the stratified anisotropic materials is presented in Fig. 7a. Such device should transmit a wave from medium 1 into medium 2 in the negative direction of the y-axis (from the input to the output) and not transmit in other directions. In particular, there is the matched load for the above described frequency detector. We will call such devices the transmitting optical devices (which operate for the wave transmission).

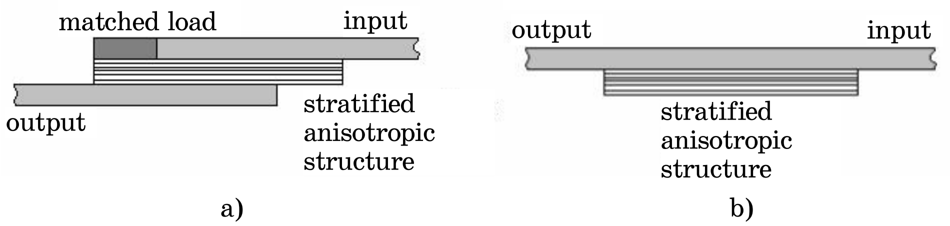

Fig. 7 *– Principle of the construction of optical devices: with transmission (a); with reflection (b)*

The second variant of the construction of the described optical devices is shown in Fig. 7b. In such devices wave should pass without damping along the structure in medium 1 and should not pass in other directions. In particular, by the matching parameters one can achieve that reflection coefficient in the directions 2 and 4 was equal to 1, and in the direction 3 was negligible. We will call such devices the reflecting optical devices (which operate for the wave reflection).

5. CONCLUSIONS

In this work, phenomena at propagation of a plane harmonic wave parallel to the stratified anisotropic structure with arbitrary direction of the anisotropy axis are considered. Investigation of the wave penetration effect into anisotropic medium with losses is carried out. Presence of the penetration effect implies the possibility of the excitation of a bulk wave in anisotropic medium by a surface incidence wave.

Questions of the practical application of the investigated structures are considered. To this end, different variants of the wave incidence are studied. Obtained results imply the possibility of the application of treated structures for the development of new optical devices.

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